KNOT THEORY: A SEMINAR COURSE IN TOPOLOGY

Instructor: Dr.Neslihan Gügümcü
Contact Info: neslihan.gueguemcue@mathematik.uni-goettingen.de, Office No: 301
Prerequisites for the course: Linear Algebra, Algebra
Instruction Language: English
Course Time: Summer Semester 2020 Friday Class Hour will be announced
Place: -

Description: Topology (ancient greek; $\tau o\pi o\varsigma + \lambda o\gamma o\varsigma$) is a field of mathematics that studies geometrical objects by considering them made of rubber and thus instead of rigid measurements such as *area*, *volume* or *angle*, measurements subject to the rubber soul are the tools of topology. Topology roots back in the studies of the 19^{th} century scientists such as Gauss, Tait, Ampere, Thomson. Gauss tried to understand earth' s magnetic potential via linked curves in space, Thomson suggested that atoms were knotted vortices in aether. These studies aroused a great mathematical interest in nicely shaped curves in space so called knots and with the developments in topology in the beginning of the 20th century, the study of knots became a mathematical theory on its own. Knot theory is still one of the active areas in mathematics with many striking applications in biology (studies in DNA structure and enzymology), physics (quantum physics, Chern-Simons theory, Gauge theory), and chemistry (in molecule structure, synthesizing molecules).

In this course, we will construct the fundamentals of knot theory, learn about mathematical tools for classifying knots, investigate the physical aspects of the theory and will discuss basic notions of algebraic topology and low-dimensional topology.

This will be a seminar course and the outlined program given below will be presented by the registered students.

Course Program

- Fundamentals of Knot Theory; equivalent definitions of a knot/link, homotopy, ambient isotopy, Reidemeister Theorem, invariant of a knot
- Basic knot invariants, 3-colorability, Fox n-coloring, quandles
- Orientation on knots/links, Gauss linking number, writhe, framed knots and links, homotopy, regular homotopy and isotopy
- Classification of planar curves in the plane (Whitney-Graustein Theorem)
- Kauffman bracket polynomial and the Jones polynomial
- Chirality, reversion, mutation, Tait' s conjectures
- Knot invariants via statistical mechanics concepts (quantum invariants), tensor product, the quantum Yang-Baxter equation, A quantum model for the Jones polynomial
- Other entangled structures; tangles, braids; composition, Alexander theorem, long knots, knotoids
- Artin braid group, Markov theorem

- Braid group and its representations via diagrammatic algebras; Temperley-Lieb algebra
- Bracket for braid diagrams, trace map, construction of the Jones polynomial through TL-algebra
- Back to basics in geometric topology: surfaces, classification of surfaces, genus, Seifert surface, genus of a knot
- 3-dimensional manifolds, complement of a knot, knot group, combinatorial description of the knot group, Gordon - Luecke theorem