A SEMINAR COURSE: FUNDAMENTALS IN GEOMETRIC TOPOLOGY

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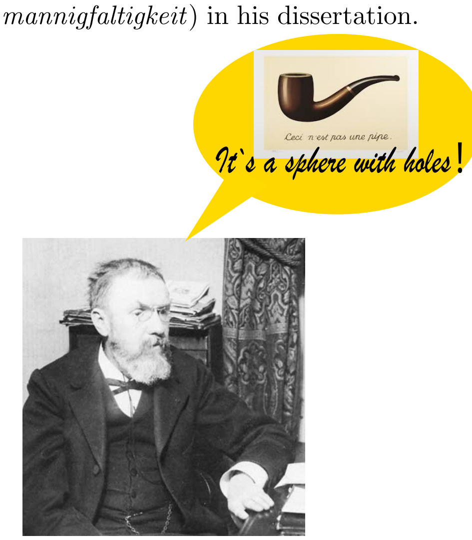
Prerequisites for the course: Linear Algebra, Algebra, Point-set Topology
Instruction Language: English
Course Time: Fall Semester 2020 – class hours will be announced

Description: This course will be the second of the online seminar courses in topology where we will cover fundamental –mostly low-dimensional–notions of geometric topology. The registered students are required to present an online seminar on the topic assigned by the instructor and attend 90% of the lectures.

Background: Abstract spaces have been studied in their own right with the use of geometric and topological tools for more than two centuries. Gauss was the first to consider surfaces and their intrinsic geometric properties without any reference to an ambient space that they lie in: With his *Theorema Eggregium*, we know that curvature is an intrinsic property of a surface.

In the latter years, Riemann generalized the idea of surfaces to higher dimensions by introducing the notion of a manifold (*mannigfaltigkeit*) in his dissertation.



Henri Poincare

By the end of the 19^{th} century Poincare had already discovered some fundamental topological notions to classify manifolds such as homology, Betti numbers, homotopy and fundamental group. His conjecture on 3-dimensional manifolds that was one of the Millennium Problems: *'Every simply connected, closed* 3*-manifold is homeomorphic to the* 3*-sphere'* remained open for almost a century and was solved just in 2006 by Perelman.

The consideration of geometric topology as a separate field of mathematics began in the 20^{th} century with the work of Reidemeister on Lens Spaces. Since then, geometric topology is an active field of mathematics, continuously developing geometric and topological tools to understand mathematical spaces and maps between them in low and higher dimensions.

Course Outline:

- Topological manifolds, Smooth manifolds, Smooth maps, Manifolds with boundary [1]
- Riemannian manifolds[1, 12]
- Surfaces, classification of surfaces, triangulation, Euler characteristic, cell structure[1]
- Basic notions in algebraic topology: Homotopy, isotopy, Fundamental Group; Seifert-Van Kampen Theorem [2,3]
- Covering Spaces, group actions
- Homology, computations [2,3]
- Immersions, submersions, embeddings, submanifolds, Whitney Embedding Theorem, Knots and links [1, 4, 5, 6]
- Jones polynomial of knots [4, 5]
- 3-manifolds, Heegaard splittings and diagrams, homology spheres, Lens spaces [5]
- homeomorphisms of surfaces, Dehn-Lickorish Theorem [5,7,8,10, 11]
- Surgery of 3-manifolds, Framed knots/links, Kirby calculus, Kirby moves, Lickorish-Wallece Theorem [4,5,8]
- Invariants of 3-manifolds via knot invariants [4]

Some of the Reference Books&Papers

[1]John M.Lee, Introduction to Smooth Manifolds

[2]Allen Hatcher, Algebraic Topology

[3]William Massey, A basic course in Algebraic Topology

[4] Louis Kuaffman, Knots and Physics

[5] Prasalov&Sossinsky, Knots,Links,Braids and 3-Manifolds

[6] Knots and Surfaces, Gilbert&Porter

[7] Max Dehn (1938), *Die Gruppe der Abbildungsklassen*, Acta Mathematica, 69 (1): 135–206, doi:10.1007/BF02547712

[8] Rob Kirby (1978), *A calculus for framed links in* S^3, Inventiones Mathematicae, 45 (1): 35–56, doi:10.1007/BF01406222, MR 0467753.

[9]Roger Fenn and Colin Rourke (1979), *On Kirby's calculus of links*, Topology, 18 (1): 1–15, doi:10.1016/0040-9383(79)90010-7, MR 0528232.

[10] W.B.R.Lickorish, *A representation of orientable combinatorial 3-manifolds*, Ann.of Math.(2), 76, 531-540

[11]Max Dehn, *Über die Topologie des dreidimensionalen Raumes*, Ann.ofMath., 69, 1910, 137-168

[12]Manfredo do Carmo, Riemannian Geometry, Basel, Birkhäuser