Braidoids

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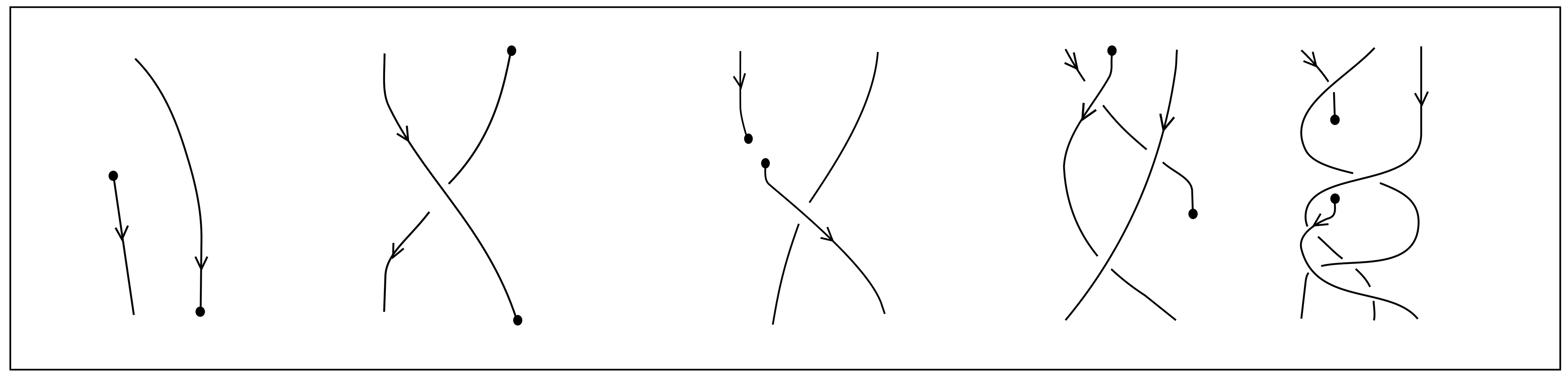
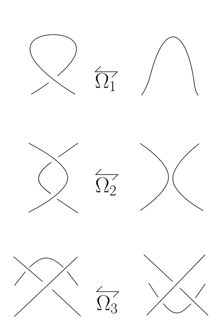


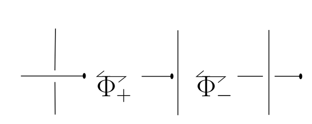
Knotoids

Knotoid diagrams are open knot diagrams with two distinct endpoints that may appear in any region of the diagram [4, 1].

- A *knotoid* is an equivalence class of all equivalent knotoid diagrams under the relation induced by the Ω-moves, shown in Figure 1a.
- The theory of knotoids in S^2 extends the classical knot theory.
- There is an inclusion map from the set of knotoids in $\mathbb{R}^2$ in to the set of knotoids in S^2, induced by the inclusion $\mathbb{R}^2 \hookrightarrow S^2$.



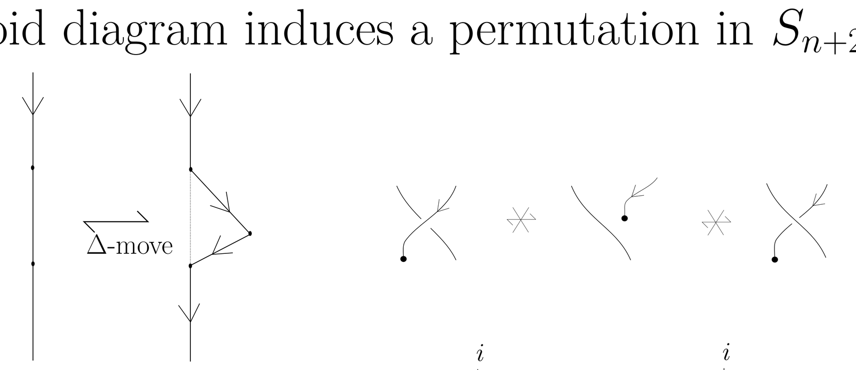
Together with isotopy of S^2, Ω- moves generate an equivalence relation on knotoid diagrams

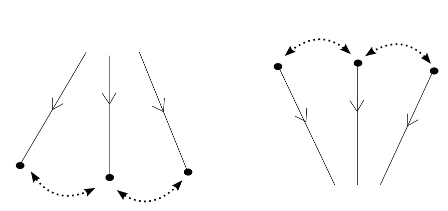


It is forbidden to pull an endpoint over/under a transversal strand!

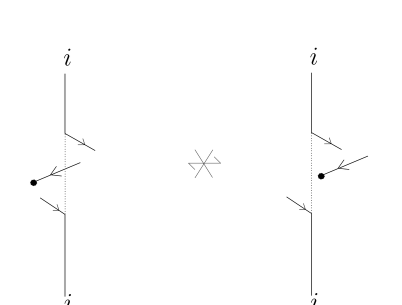
Braidoids & Moves on Braidoids

A *braidoid diagram* with $n+2$ strands is a diagram in the plane with n braid strands, and two *free strands* containing the *free ends*. The *corresponding ends* of strands are numerated with an integer $n \in \{1, 2..., n+1\}$. The free ends are denoted by l and h. With the identification, $l = h$, the endpoints a braidoid diagram induces a permutation in S_{n+2}.





A Δ-move and swing moves of the free ends

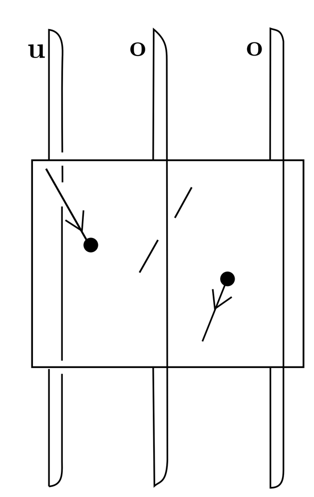


It is forbidden to pull an endpoint over/under a transversal strand also across the vertical alignment of corresponding ends!

The end-to-end Closure

The aim is to obtain a knotoid diagram from a braidoid diagram.

1. Given a braidoid diagram B. Label top of each corresponding ends of B with either o or u, where o stands for over and u stands for under.
2. Connect the corresponding ends with a simple arc which goes entirely over or under every strand it meets, accordingly to the label of the top end.



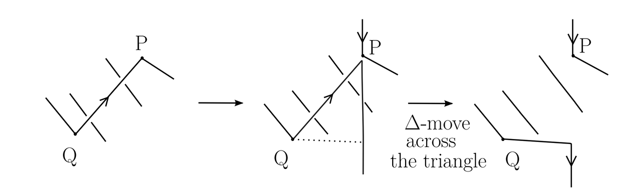
The closure of an abstract labeled braidoid diagram

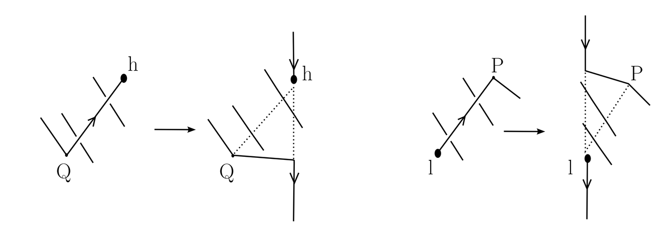
A *braidoid* is an equivalence class of braidoid diagrams under the equivalence relation induced by Δ-moves and swing moves.

Braidoiding Algorithm

The aim is to turn a knotoid diagram into a braidoid diagram. For this, we eliminate the *up-arcs* by the following algorithm.

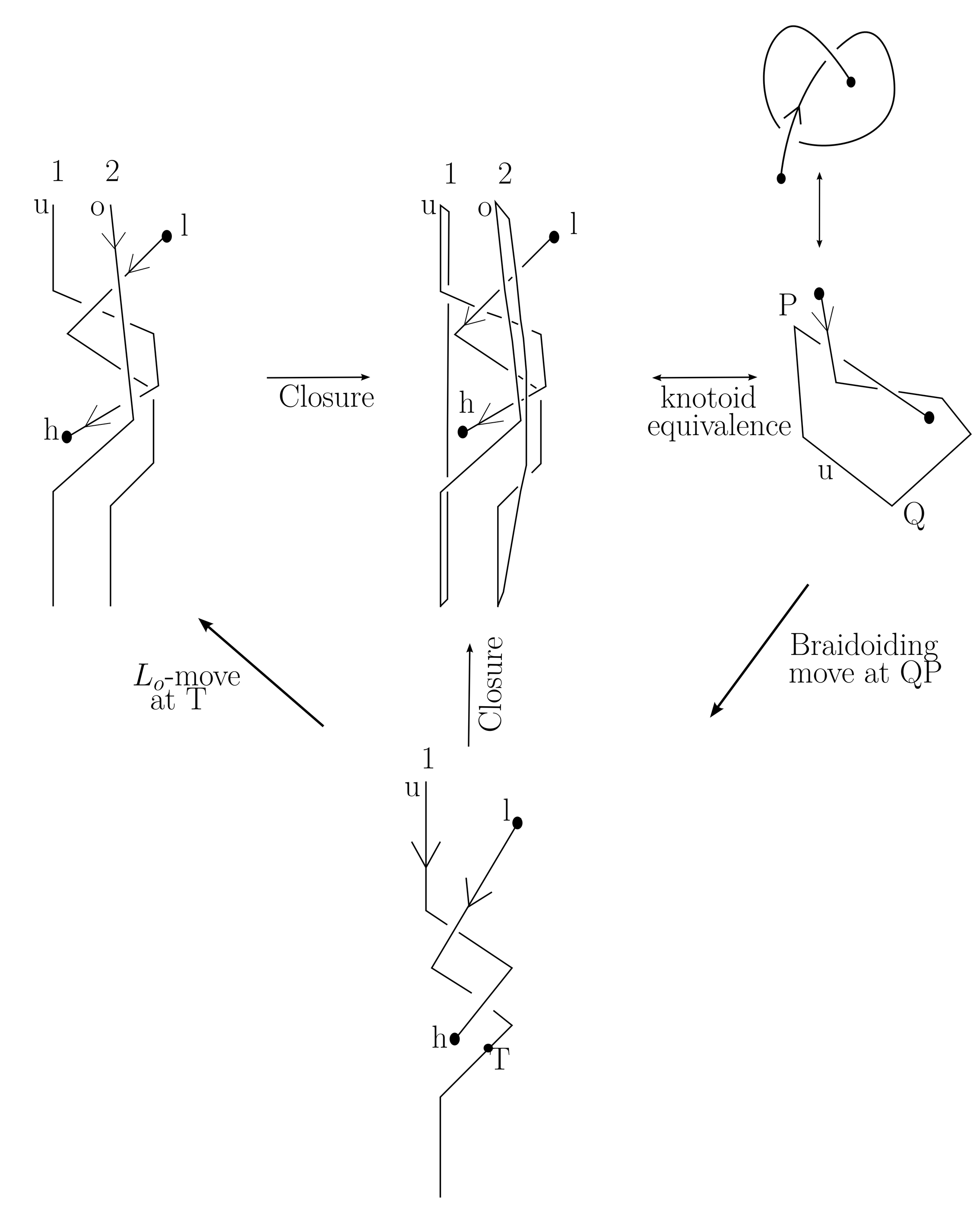
1. Subdivide each arc that is sloped upwards so that each contains at most one type of crossings, either over or under-crossings, or no crossings at all. The smaller arcs are called *up-arcs* of the diagram.
2. Cut each up-arc at a point, and pull the resulting strands to top and bottom ends.
3. The resulting strands are pulled entirely over or under the rest of the diagram, accordingly to the type of crossing of the initial up-arc.
4. Turn the resulting strands into braidoid strands via Δ-moves.
5. The resulting braidoid strands are labeled with the label of the initial up-arc.





Up-arcs and elimination of them

Example

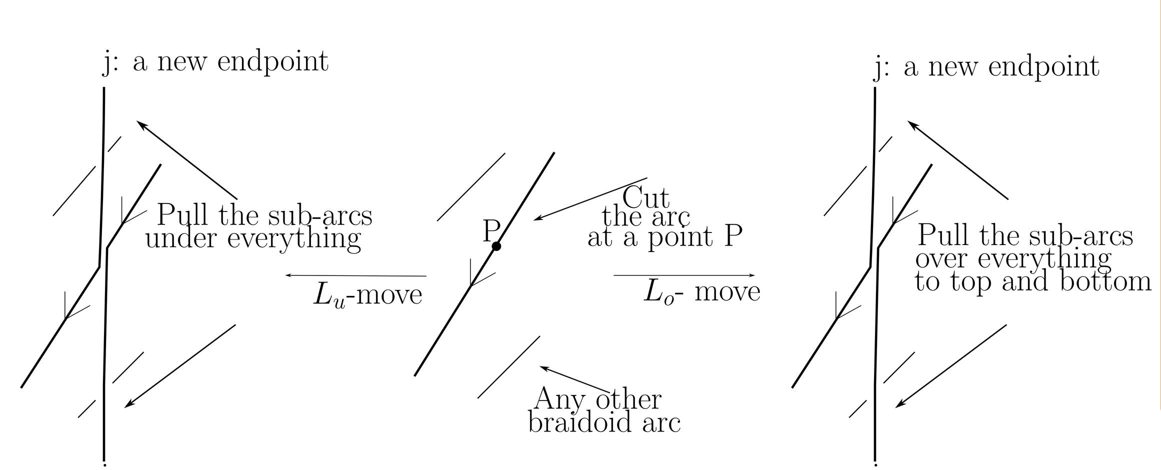


An analog of Alexander's theorem for knotoids

Any knotoid in $\mathbb{R}^2$ is equivalent to the (end-to-end) closure of a braidoid.

L-equivalence of knotoids

L-moves are defined braidoid diagrams.



L-moves consist of L_o- and L_u-moves

Two braidoid diagrams are L-*equivalent* if they are related to each other by a finite sequence of L-moves with Δ-, and swing moves.

An analog of Markov's Theorem for knotoids

The (end-to-end) closures of two braidoid diagrams are equivalent knotoids if and only if the braidoid diagrams are L-equivalent.

References

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